Abstract
We consider a malaria transmission model with SEIR (susceptible-exposed-infected-recovered) classes for the human population, SEI (susceptible-exposed-infected) classes for the wild mosquitoes and an additional class for sterile mosquitoes. We derive the basic reproduction number of infection. We formulate an optimal control problem in which the goal is to minimize both the infected human populations and the cost to implement two control strategies: the release of sterile mosquitoes and the usage of insecticide-treated nets to reduce the malaria transmission. Adjacent equations are derived and the characterization of the optimal controls are established. Finally, we quantify the effectiveness of the two interventions aimed at limiting the spread of Malaria. A combination of both strategies leads to a more rapid elimination of the wild mosquito population that can suppress Malaria transmission. Numerical simulations are provided to illustrate the results.

SEIR (Susceptible-Exposed-Infected-Recovered) classes for Human
SEI (Susceptible-Exposed-Infected) classes for Wild Mosquitoes

Theorem
Theorem 1: If \( b < b^* \), there exists a disease-free equilibrium in system (1). The disease-free equilibrium of the state system is locally asymptotically stable when \( R_0 < 1 \), and unstable when \( R_0 > 1 \).

Optimal Control
Two controls: \( u_1(t) \): the efficacy of the bed net usage, and \( u_2(t) \): the rate of releasing sterile mosquitoes. Our goal is to determine an optimal control pair \((u_1(t), u_2(t))\) that minimizes the objective functional:

\[
J = \int_0^T \left( w_1 \lambda + \frac{1}{w_2} (\psi_v^2 + \bar{w}_v^2) \right) dt.
\]

The state system with two controls is given by

\[
\begin{align*}
\frac{dS_H}{dt} &= \lambda_H + \psi_H N_H + \rho_H R_H - \lambda_H(t) S_H - f(H) N_H S_H, \\
\frac{dE_H}{dt} &= \lambda_H(t) S_H - (1 - u_1(t)) \lambda_E(t) S_E - f(H) N_H E_H, \\
\frac{dI_H}{dt} &= \lambda_E(t) S_E - \psi_E E_E - f(H) N_H I_E, \\
\frac{dR_H}{dt} &= \psi_E E_E - \lambda_R - f(H) N_H R_H, \\
\frac{dS_V}{dt} &= \psi_v N_V - \lambda_v(t) S_V - f_V(N_V) S_V, \\
\frac{dE_V}{dt} &= \lambda_v(t) S_V - \psi_v E_V - f_V(N_V) E_V, \\
\frac{dI_V}{dt} &= \psi_v E_V - f_V(N_V) I_V, \\
\frac{dR_V}{dt} &= \psi_v E_V - f_V(N_V) R_V.
\end{align*}
\]

Characterization of the optimal controls:

\[
\begin{align*}
u_1^* &= \min \left( \max(0, m_1), M \right) \\
u_2^* &= \min \left( \max(0, -\bar{b}_v/w_v), M \right)
\end{align*}
\]

where

\[
m_1 = \frac{\lambda_H - \lambda_v}{\sigma_v N_v + \sigma_E N_E}.
\]

Table 1: Description of model parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \nu_h )</td>
<td>Insecticide rate of human equilibrium.</td>
</tr>
<tr>
<td>( \nu_m )</td>
<td>Insecticide rate of mosquito equilibrium.</td>
</tr>
<tr>
<td>( m_1 )</td>
<td>The maximum number of mosquitoes to be killed per unit time.</td>
</tr>
<tr>
<td>( m_2 )</td>
<td>The term in the objective function.</td>
</tr>
<tr>
<td>( \bar{b}_v )</td>
<td>The term in the objective function.</td>
</tr>
<tr>
<td>( \sigma_H )</td>
<td>Transmission probability from an infective human to a susceptible mosquito if contact (bite) occurs.</td>
</tr>
<tr>
<td>( \sigma_V )</td>
<td>Transmission probability from a recovered human to a susceptible mosquito if contact (bite) occurs.</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Per capita birth rate of mosquitoes.</td>
</tr>
<tr>
<td>( \psi_v )</td>
<td>Transmission probability from an infected mosquito to a susceptible mosquito if contact (bite) occurs.</td>
</tr>
<tr>
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<td>Transmission probability from an infected mosquito to a susceptible mosquito if contact (bite) occurs.</td>
</tr>
<tr>
<td>( \lambda_v )</td>
<td>Per capita death rate of mosquitoes.</td>
</tr>
<tr>
<td>( \beta )</td>
<td>The basic reproduction number of infection.</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>The basic reproduction number of infection.</td>
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</tbody>
</table>

Numerical Simulations

Optimal Two Controls: (1) ITN

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